## ■ Rozdzielenie zmiennych w PDE

In[1]:=
```
tr1 = w[x, t] → ϕ[x] * ψ[t]
```

Out[1]=
$$w[x, t] \to \phi[x]\, \psi[t]$$

In[2]:=
```
pde1[u_] = D[u[x, t], t] == a * D[u[x, t], x, x]
```

Out[2]=
$$u^{(0,1)}[x, t] == a\, u^{(2,0)}[x, t]$$

In[3]:=
```
eq2 = Expand[pde1[w]]
```

Out[3]=
$$w^{(0,1)}[x, t] == a\, w^{(2,0)}[x, t]$$

In[4]:=
```
eq3 = eq2 /. tr1 /. D[tr1, t] /. Table[D[tr1, {x, i}], {i, 1, 2}]
```

Out[4]=
$$\phi[x]\, \psi'[t] == a\, \psi[t]\, \phi''[x]$$

In[5]:=
```
eq4 = PowerExpand[eq3]
```

Out[5]=
$$\phi[x]\, \psi'[t] == a\, \psi[t]\, \phi''[x]$$

In[9]:=
```
eq4 / (ϕ[x] * ψ[t])
```

Out[9]=
$$\frac{\phi[x]\, \psi'[t] == a\, \psi[t]\, \phi''[x]}{\phi[x]\, \psi[t]}$$

ale

In[7]:=
```
eq5 = Thread[eq4 / (ϕ[x] * ψ[t]), Equal] // Expand
```

Out[7]=
$$\frac{\psi'[t]}{\psi[t]} == \frac{a\, \phi''[x]}{\phi[x]}$$

In[10]:=
```
FullForm[eq4 / (phi[x] * psi[t])]
```

Out[10]//FullForm=
```
Times[Equal[Times[\[Phi][x], Derivative[1][\[Psi]][t]],
  Times[a, \[Psi][t], Derivative[2][\[Phi]][x]]], Power[phi[x], -1], Power[psi[t], -1]]
```

## ■ Jak dziala Thread?:

In[11]:=
```
Thread[Log[x == y], Equal] (* tam gdzie spotkasz operator ==,
to lewy i prawy argument zamien na logarytmy.*)
```

Out[11]=
$$\text{Log}[x] == \text{Log}[y]$$

In[23]:=
```
FullForm[a * b^c]
Thread[Exp[a * b^c], Times]
```

Out[23]//FullForm=
```
Times[a, Power[b, c]]
```

Out[24]=
$$e^{a+b^c}$$

Zatem mozemy rozdzialeac zmienne:

In[25]:=
```
Times[Equal[z * x, 3], Power[b, -1]]
Thread[Times[Equal[z * x, 3], Power[b, -1]], Equal]
```

Out[25]=
$$\frac{x\, z == 3}{b}$$

Out[26]=
$$\frac{x\, z}{b} == \frac{3}{b}$$

wracajac do tematu

In[27]:=
```
eq6 = Map[Simplify, eq5]
```

Out[27]=
$$\frac{\psi'[t]}{\psi[t]} == \frac{a\,\phi''[x]}{\phi[x]}$$

In[32]:=
```
sol = {DSolve[eq6[[1]] == c, ψ[t], t], DSolve[eq6[[2]] == c, ϕ[x], x]} // Flatten
FullForm[%]
```

Out[32]=
$$\left\{\psi[t] \to e^{c\,t}\,C[1],\ \phi[x] \to e^{\frac{\sqrt{c}\,x}{\sqrt{a}}}\,C[1] + e^{-\frac{\sqrt{c}\,x}{\sqrt{a}}}\,C[2]\right\}$$

Out[33]//FullForm=
```
List[Rule[\[Psi][t], Times[Power[E, Times[c, t]], C[1]]], Rule[\[Phi][x], Plus[Times[
    Power[E, Times[Power[a, Rational[-1, 2]], Power[c, Rational[1, 2]], x]], C[1]], Times[
    Power[E, Times[-1, Power[a, Rational[-1, 2]], Power[c, Rational[1, 2]], x]], C[2]]]]]
```

In[34]:=
```
Cases[%, _[ x_, y_] -> y] (* jesli napotkasz jakis dwuargumentowy operator,
zamien go na jego drugi argument *)
```

Out[34]=
$$\left\{e^{c\,t}\,C[1],\ e^{\frac{\sqrt{c}\,x}{\sqrt{a}}}\,C[1] + e^{-\frac{\sqrt{c}\,x}{\sqrt{a}}}\,C[2]\right\}$$